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A STUDY ON INVENTORY CONTROL SYSTEM FOR A SUPPLY CHAIN USING MARKOV DECISION PROCESSES

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ABSTRACT

The necessity of either strategic or emergency planning is attracting an ever-increasing amount of focus, and this trend holds true regardless of the type of industrial output being undertaken. This is especially true when taken into consideration in light of the increasing degree of rivalry that exists in the markets of today. The management of companies consistently works towards the goal of implementing policies that, over the course of time, will minimise the total amount of money that is spent on operational expenditures. In this thesis, an inventory control problem is examined via the lens of a case study, and an optimal stationary method is proposed as a solution. The problem that has to be solved is figuring out how to time the production of limestones at a firm that is placed in the greatest possible way. We are seeking for a production plan that will bring the estimated total of discount charges down to a more manageable level. For the sake of this specific piece of study, a Markov decision process with an infinite horizon served as the basis for the mathematical modelling framework. The issue is reframed as a problem that requires sequential judgements, and it is then subjected to a battery of rigorous computer tests in order to determine how to proceed. We created a recommendation for a first policy, which was obtained through the policy iteration process, and at this time we are deciding whether or not it is the most viable choice. On the basis of these considerations, our policy approach is regarded as the most viable alternative. Despite the fact that this policy is the best available choice, we do not make the claim that it is the one and only optimal solution to the problem that has been discussed in this article. It's conceivable that there are more sorts of optimal rules out there that can help you solve this challenge.

Keywords: *industrial, markets, production*

INTRODUCTION

The management of the flow of goods and services, which starts at the point of production and continues all the way to the point of consumption, is what we mean when we talk about the supply chain (retailer). Movement and storage of raw materials that are involved in work progress, inventories, and completely furnished items are all a part of this process. The intricacy of the supply chain may vary substantially from industry to industry due to the fact that it is present in both service and manufacturing businesses. Nonetheless, supply chains are present in both. Because inventories exist at each and every level of the supply chain, whether as raw materials, semi-finished items, or finished goods, the choice about inventories is a crucial component of the management of the supply chain. They are also capable of functioning as Work-in-process between the various phases or stations. The effective management of inventories is essential to the functioning of supply chain activities since the cost of retaining stocks can range anywhere from 20% to 40% of their worth.

The typical goal of a multi-echelon inventory model is to coordinate the inventories at the various echelons in order to reduce the overall cost that is connected with the multi-echelon inventory system as a whole. It is also possible for this to be an appropriate purpose in situations when some strata of the organisation are handled by either its suppliers or its merchants. The reason for this is that one of the fundamental tenets of supply chain management is that a business ought to make it a priority to cultivate informal partnership relationships with its retailers and suppliers, so that all parties involved can collectively achieve the highest possible level of overall profit. It would be accurate to argue that information technology is an essential component of supply chain management. [Citation needed] [Citation needed] The introduction of brand-new items to the market can now be accomplished in a matter of seconds thanks to developments in technology, which has led to an increase in the demand for these goods. Let's spend some time discussing the importance of information technology in the management of supply chains. Several academics have looked at the multi-echelon inventory system, and the results of their findings about its usefulness in supply chain management have been published relatively recently. As a result of the integration of several operators into the network, supply chains are able to minimise the overall costs involved without reducing the effectiveness of their client service. In the 1980s, continuous review models of multi-echelon inventory systems focused more on repairable goods in a Depot-Base system than they did on consumable products.

Repairable products and batch ordering are common themes throughout all of these concepts. An approximation of the inventory structure in SC was suggested by Sven Axsater in his model. Sherbrooke published a fundamental and important article in the subject of continuous review multi-echelon inventory system in 1968. This study is considered to be one of the earliest publications in the field. He estimated that the (S-1,S) policies that were in place in the Depot-Base systems for repairable products in the American Air Force, and he was able to come up with an approximation of the typical inventory and stock out level at bases. Seifbarghy and Jokar performed an analysis on a two-tiered inventory system that consisted of one warehouse and many retailers and was managed using a continuous review (r,Q) strategy. Benita M. Beamon offered a comprehensive evaluation in her contribution (1998). It is crucial to note that a significant amount of research in this field is based on the groundbreaking work that Clark and Scarf did many years ago. These models were primarily responsible for the development of the supply chain idea (1960). The majority of models assume immediate supply of order when dealing with continuous review perishable inventory models with random lives for the goods. As a result of the additional complexity that is introduced into the analysis of these models as a result of the assumption of positive lead times, there are only a select few models that can cope with positive lead times. Elango investigated a continuous review perishable inventory system at Service Facilities as part of their research (2001). A policy of continual reviews and continuous surveys with positive lead times in a two-echelon structure. The Supply Chain was something that K.Krishnan and C.Elango thought about. A relatively unexplored new field in supply chain management is that of service facilities in the inventory. In this chapter, we discussed an inventory system model that is kept in service facilities at Retailer vendors as part of a tandem supply chain that also includes Distribution Centers and Retailer Vendors (DC). RV uses one of its inventory items in order to provide service to the consumer. In the Retail Vender node, the (s, S) policy has been adopted for the purpose of inventory replenishment.

The vast majority of manufacturing businesses are structured as networks of production and distribution locations, which are responsible for the acquisition of raw materials, the transformation of those materials into completed items, and the delivery of those finished goods to end users. When referring to manufacturing and distribution networks, the word "multi-echelon" or "multi-level" can also be used interchangeably. This is because these terms refer to the same thing: networks (or supply chains) in which an item goes through more than one stage before reaching the end client. All throughout the supply chain, inventories may be found in a variety of formats and for a variety of purposes. They are present in the distribution warehouses, and they are also present "in-transit" or "in the pipeline" along each route that connects these facilities to one another. There is a connection between all of these in the sense that:

- The downstream sites create demands on the upstream inventories.
- These uncertain demands, combined with uncertain production and / or transit times largely determine the inventory at a given site.

The central premises here is that the lowest inventories result when the entire supply chain is considered as a single system. Such co-ordinated decisions have produced spectacular results at, which were able to reduce their respective inventory levels by over 25%, consequently reduced the cost by 33%.

Motivation and problem

Setting The process of multi-echelon inventory management, which is often referred to as effective inventory maintenance in a supply chain, is one that takes a lot of time to complete. From the beginning of the twenty-first century, a large number of academics have concentrated their research in this general area, with the intention of producing a model for inventory control that is universally acknowledged. Despite this, there is not yet a complete model that can be used to regulate inventory across a number of different echelons. The majority of the systems that were considered made the assumption that the demand rate and the lead time are either fixed or follow a well-known probability distribution, which is a more constrained requirement. This assumption was made by the majority of the systems that were taken into consideration. The early researchers who worked in less stressful environments and concentrated on demand and replenishment time provided as a source of inspiration for the authors. The following organisational pattern may be found across this entire book. In this first section, we are going to discuss the essential features of the problem. The second part of the article provides a clear and concise explanation of the thought process that led to the formulation of the problem that was presented. The relevant previous research is analysed and addressed in the third section. In Section 4, both the model and the technique that will be used to solve the problem are provided. In Part 5, we talk about the policy iteration approach, which is designed to find the optimal value for the policy parameter and validate the model that was described in Section 4. In Section 6, you'll find a presentation of the numerical examples, together with an analysis of the method that works best to solve them.

Objective of the study

- To study on Inventory Control System for a Supply chain using Markov Decision processes.
- To study on a supply chain consists of various levels.

LITERATURE REVIEW

The model presented here draws inspiration from two distinct bodies of research: multi-echelon inventory systems and the Markov decision process for inventory control system.

Multi-echelon inventory systems

Because a supply chain is made up of several tiers or levels of strata, one of the most significant questions that needs to be addressed is how partners at the various levels of a supply chain interact with one another. This is a question that has to be answered because it is crucial. Both the "push" system and the "pull" system leave traces in the existing literature, but the two approaches are quite distinct from one another and cannot be confused with one another in any way. In a distribution system such as the one described in this paper, a push philosophy would mean that there is a central decision maker, such as a warehouse manager, who has access to information about inventory levels at all of the facilities that are of concern; all decisions regarding inventory are then made centrally based on the information that is gathered from these facilities. In other words, a push philosophy would mean that a distribution system uses a pull philosophy. On the other hand, the pull system gives local managers the authority to make decisions on inventories, and these managers are required to take into account the specifics of the environments in which they work .Both the push and pull systems contain decision criteria (order quantity, reorder point, etc.) that are configured in such a way as to minimise the overall system costs. This is done in order to maximise efficiency. Due of the discussion of a continuous review pull system in this article, we will confine our literature study to works that concentrate on systems similar to those discussed in the article. For a more in-depth investigation of the push mechanism, the reader is referred to Federgruen which may be found here.

It is believed that Sherbrooke developed one of the earliest versions of the continuous review multi-echelon inventory model. He is considering a structure with two levels, the first of which would be comprised of a variety of retail outlets, and the second of which would be a distribution centre that would supply those outlets. The now-famous METRIC approximation was conceived by him while he was working to determine how much stock should be in the system at any one moment. When the demand distribution is Poisson, the Poisson distribution may be easily described as the number of unfulfilled orders from retailers by employing the METRIC approximation. Graves develops the METRIC method further by estimating two parameters to reflect the unfulfilled retail orders. The mean and the variance are the two parameters that are being discussed here. He uses the negative binomial distribution to these parameters in order to determine which inventory management technique will be most effective for the given situation. later presents a precise solution to the problem and demonstrates that the Graves two-parameter approximation overestimates the retailer backorders, while the METRIC approximation underestimates the number of backorders placed by retailers. This was done by proving that the Graves approximation overestimates the number of backorders placed by retailers. provide basic enhancements to the Sherbrooke principle that is at its core.

All the above studies use a one-for-one ordering policy,

Simply due to the fact that there has been a demand for it. Axsater provided an illustration of how the processes for the one-for-one ordering policy can be altered in such a way that they continue to be applicable even when there is only one store. An study of the batch-ordering rules in arborescent systems (where the number of merchants is higher than 1) may be carried out in a way that is comparable to that of Sherbrooke. It's possible that Deuermeyer and Schwarz were the first persons to carry out a study of such a complex system. They created an estimate of the mean and variance of the lead time demand in order to calculate the normal inventory levels and backorders at the warehouse. This was done by making an estimate of the mean and variance of the lead time demand. The assumption that the lead time demand follows a normal distribution was used in the process of carrying out these steps. In addition to giving a summary of the relevant literature, please explain on the many ways in which the model might be improved.

examination of sequential decision making based on probabilistic models with an open-ended time frame for future planning. Both the Markov process and dynamic programming were important factors in the creation of this method. In the early 1950s, Bellman [2] was the one who came up with the idea for the paradigm known as dynamic programming. In dynamic programming, the essential principles that are involved are the states, the principle of optimality, and functional equations. important concept that was taken from Markov chains and dynamic programming and was utilised in the construction of a policy iteration algorithm for the solution of probabilistic sequential decision making processes that had an unbounded planning horizon. The Markov decision model may be useful in a number of settings, including as the management of supply chains, the performance of preventative maintenance, the production of goods, and even the operation of telecommunications networks. did an analysis to establish the inventory levels that would result in the most efficient management of service rate at a facility that delivers services in order to determine the inventory levels that would result in the most efficient control of service rat.

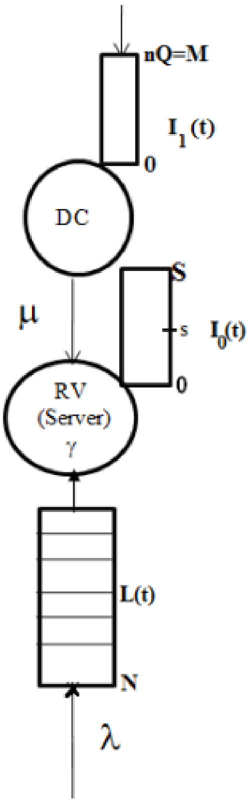
Model development and solutions

The purpose of our model is to provide a near optimal ordering policy and order quantity (reorder point). i.e., (s, Q) – type for retailers and (0,S)-type for warehouse that minimizes the total logistic and maintenance costs subject to customer service constraints. Three subsystems need to be analyzed.

- Inventory at each retailer.
- The demand process of the warehouse.
- The inventory at the warehouse. A synthesis of these subsystems is used to arrive at the final

Model Formulation:

We define a Supply Chain system to be one that includes a Distribution Center (DC), a Retail Vendor (RV), and a Service Facility, with Inventory being Maintained at Each of the DC and RV Nodes. An item is only given following an exponentially long service period with a parameter of for each and every demand placed at the retailer node (RV). There is a maximum capacity of N people in the waiting area of the retailer node. When a new client arrives and sees that there are already N customers in the system, they depart. When the inventory level hits the prefixed level s, orders for Q=S-s>s items are placed. Lead time is exponentially distributed with parameter p. The inventory policy that is used at the RV node is of the (s, S) type.⌈(0) > . Demand at RV node follows a Poisson process with parameter ⌊(0) > . At DC, items are packed as Q items in one pocket with maximum inventory level nQ (n pockets). The ordering policy at DC is of (0,nQ) type where the inventory level reach 0, instantaneous replenishment nQ= M items is made. Deterministic Markov Decision policy is used solve the problem of MDP.



Let I0(t) and L(t) denote the inventory level and the number of customers in the system at time t. Then $\{(I_0(t), L(t)) : t \geq 0\}$ is a finite two-dimensional stochastic process with state space $E_1 \times E_2$,

Where $E_1 = \{0, 1, 2, \ldots, S\}$ and $E_2 = \{0, 1, 2, \ldots, N\}$.

Decision Sets:

The reordering decisions taken at each state of the system $(i,q) \in E$, where, $I(t) = i$ and $X_0(t) = q$.

Let Ai (i =1, 2, 3) denotes the set of possible actions where, $A_1 = \{0\}$, $A_2 = \{0, 1\}$,

$A_3 = \{2\}$, $A = A_1 \cup A_2 \cup A_3$, 0 1 means reorder for 'Q = S-i' items at level i and 2 means compulsory order for S items when inventory level is zero.

Suppose Ð denote the class of all stationary policies, then a policy f (sequence of decisions) can be defined as a $f: E \rightarrow A$,

$$f(i,q) = \begin{cases} \{0,1\} & 1 \le i \le s, q \in E_2 \\ \{0\} & s+1 \le i \le S \quad , q \in E_2 \\ \{2\} & i = 0, q \in E_2 \end{cases}$$

Objective of the problem is to find the optimal reorder level s so that the long run expected total cost rate is minimum.

Notations and Assumptions:

1) $E_1 \times E_2 = E$ *is the state space of the Stochastic Process* $\{(I(t), L(t)) : t \ge 0\}$,
2) Where $E_1 = \{0,1,2,,...,S\}$ *and* $E_2 = \{0,1,2,,...,N\}$
3) $A_{(i,q)}$ decision set corresponding to state $(i,q) \in E$
4) $p_{(i,q)}^{(j,r)}(a)$ the transition probability from state when action a is taken at state.
5) Inventory levels are reviewed at the time of service completion epochs.
6) Reordering policy is (s, S): $Q = S - s$ items ordered when the inventory level reaches s (prefixed level),
7) D- the class of stationary policies.

Analysis

Let R denote the stationary policy, which is time invariant and Markovian Policy (MR). From our assumptions it can be seen that $\{(I_0(t), L(t)) : t \ge 0\}$ is denoted as the controlled process $\{(I_0^R(t), L^R(t)) : t \geq 0\}$ when policy R is adopted. Since the process $\{(I_0^R(t), L^R(t)) : t \geq 0\}$ i is a Markov Process with finite state space E. The process is completely Ergodic, if every stationary policy gives raise to an irreducible Markov chain. It can be seen that for every stationary policy $f \in F, \{I_0^f, L^f\}$ is completely Ergodic and also the optimal stationary policy R* exists, because the state and action spaces are finite.

h - holding cost / unit item / unit time
c_1 – waiting cost / customer / unit time
c_2 – reordering cost / order
g - balking cost / customer
β - service cost / customer
$\bar{I}^f$ - mean inventory level
$\bar{W}^f$ - mean waiting time in system
η_a^f - reordering rate
η_b^f - balking rate
η_c^f - service completion rate

Our objective is to find an optimal policy f^* for which $C^{f^*} \leq C^f$ for every MR policy in f^{MR}
For any fixed MR policy $f \in f^{MR}$ and $(i,q),(j,r) \in E$, define

$$\Phi_{iq}^f(j,r,\mathrm{t}) = P\mathrm{r}\{I_0^f(t) = j, \mathrm{L}^f(t) = \mathrm{r} \mid I_0^f(0) = \mathrm{i}, \mathrm{L}^f(0) = q\},\ (i,q),(j,r) \in E.$$

Now $\Phi_{iq}^f(j,r,t)$ satisfies the Kolmogorov forward differential equation $\Phi_i'(\mathrm{t}) = \Phi(\mathrm{t})\mathrm{A},$ where A is an infinitesimal generator of the Markov process $\{(\mathrm{I}_0^f(\mathrm{t}), L^f(\mathrm{t})) : \mathrm{t} \geq 0\}$. For each MR policy f, we get an irreducible Markov chain with the state space E and actions space A which are finite

$$\Phi^f(\mathrm{j},\mathrm{r}) = \lim_{t \to \infty} \Phi_{iq}^f(j,r;t)$$

exists and is independent of initial state conditions.

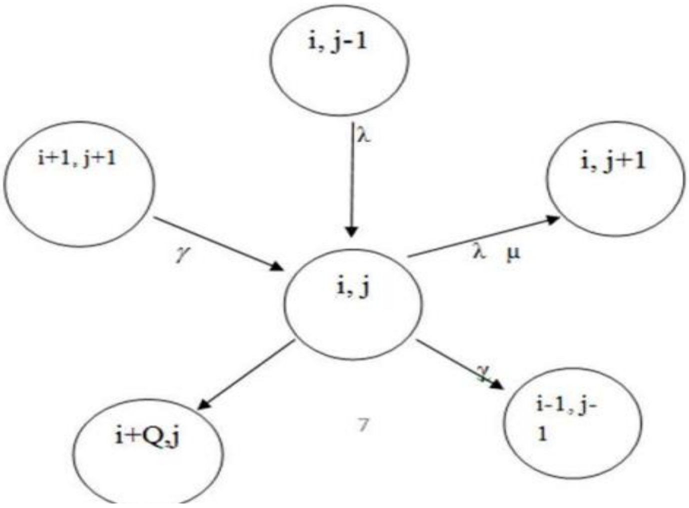


Fig (2) represent the in-rate and out-rate flow diagram of the system states.

Now the system of equations can be written in order as follows

$$\lambda\Phi^f(S,0)=\mu\sum_{j=0}^{s}p_j\Phi^f(j,0)$$

$$(\lambda+\gamma)\Phi^f(S,r)=\mu\sum_{j=0}^{s}p_j\Phi^f(j,r)+\lambda\Phi^f(S,r-1),\quad 1\le r\le M-1$$

$$\gamma\Phi^f(S,N)=\mu\sum_{j=0}^{s}p_j\Phi^f(j,N)+\lambda\Phi^f(S,N-1)$$

$$\lambda\Phi^f(j,0)=\gamma\Phi^f(j+1,1),\quad s+1\le j\le S-1$$

$$(\lambda+\gamma)\Phi^f(j,r)=\gamma\Phi^f(j+1,r+1)+\lambda\Phi^f(j,r-1),\ s+1\le j\le S-1;1\le r\le N-1$$

$$\gamma\Phi^f(j,N)=\lambda\Phi^f(j,N-1),\quad s+1\le j\le S-1$$

$$(\lambda+\mu p_j)\Phi^f(j,0)=\gamma\Phi^f(j+1,1)\ ,\ 1\le j\le s$$

$$(\lambda+\mu p_j+\gamma)\Phi^f(j,r)=\gamma\Phi^f(j+1,r+1)+\lambda\Phi^f(j,r-1),$$
$$1\le j\le s;1\le r\le N-1,$$

$$(\mu p_j+\gamma)\Phi^f(j,N)=\gamma\Phi^f(j,N-1),1\le j\le s\ ,$$

$$(\lambda+\mu p_0)\Phi^f(0,0)=\gamma\Phi^f(1,1),$$

$$(\lambda+\mu p_0)\Phi^f(0,r)=\gamma\Phi^f(1,r+1)+\lambda\Phi^f(0,r-1),\quad 1\le r\le N-1\ ,$$

$$\mu p_0\Phi^f(0,N)=\lambda\Phi^f(0,N-1)$$

The above set of equations together with the condition $\sum_{(j,r)\in E}\Phi^f(j,r)=1$ determine the steady-state probabilities uniquely.

CONCLUSION

In the vendor node, the ordering policy that we use in our case is a (s, S) ordering policy. Studying policies such as one-to-one ordering, systems, or any other type of fixed ordering policy is possible using approaches that are analogous to those used. In order to evaluate the issue, we made use of the tools provided by Semi-Markov decision processes, and a linear programming strategy was applied in order to determine the most appropriate ordering level. The establishment of the inventory control policy that will help to smooth out the implement supply chain is the most important contribution that this chapter makes. This model's Markov decision process (MDP) makes use of the randomized Markov policy, which is making its debut appearance for MDP application in inventory management systems.

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